On a strategic motivation of tacit collusion: the Nash-2 equilibrium concept

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Why to bother about extending the Nash equilibrium concept?

- It does not always exist in a number of games widely used in economics:
  - Price game in the Hotelling linear city mode
  - Tullock contest
- It leads to inadequate game situation.
  - Prisoner's dilemma
  - Bertrand paradox
  - Hotelling minimum differentiation principle

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We seek for a compromise between fully myopic behavior (NE) and perfect rationality (Folk theorem).

Some existing refinements of NE

- Rationalizable conjectural equilibrium (Rubinstein and Wolinsky, 1994)
- Oligopolistic equilibrium (D'Aspremont, Dos Santos and Gerard-Varet, 2003)
- Reflexive games (Novikov and Chkhartishvili, 2003)
- Equilibrium in secure strategies (ESS) (Iskakov and Iskakov, 2005)
- Cooperative equilibrium (Halpern and Rong, 2010)
- Farsighted pre-equilibrium (Jamroga and Melissen, 2011)
- A number of concepts for cooperative games (von Neumann-Morgenstern stable set, Harsanyi's indirect dominance of coalition structures, solution in threats and counter-threats, etc.)

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Nash-2 equilibrium

Definition (profitable secure deviation)

A deviation s_i' of player i at strategy profile $s = (s_i, s_{-i})$ is profitable and secure if $u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$ and for any strategy s_{-i}' of player $-i$ such that $u_{-i}(s_i', s_{-i}') > u_{-i}(s_i', s_{-i})$

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Proposition (A. Iskakov & M. Iskakov, 2012)

$$NE \subset ESS \subset NE\text{-}2$$

Example: Prisoner's dilemma

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But! Both mutual defection and mutual cooperation are NE-2.

Bertrand model

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$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - m_c)D, & \text{if } p_i < p_{-i}, \\ (p_i - m_c)D/2, & \text{if } p_i = p_{-i}, \\ 0, & \text{if } p_i > p_{-i}. \end{cases}$$

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If the number of firms increases from one to two, the equilibrium price decreases from the monopoly price to the competitive price and stays at the same level as the number of firms increases further.

This is not very realistic: pricing above marginal cost is typical for the markets with a small number of firms.

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How to choose among multiple equilibria?

Wiseman (2014), D'Aspremont et al. (2003)

Hotelling model of linear city with symmetric locations

The «linear city» Hotelling model

Location is the distance $d \in [0; 1]$ between firms 1 and 2 equidistant from the ends of the line.

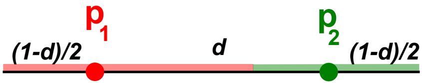

Fig.1

Consumers are uniformly distributed. Demand is totally non-elastic. Transportation costs are linear.

Price-setting game

Profit functions of firms $i = 1, 2$:

$$\pi_i(p_i, p_{-i}) = \begin{cases} p_i(1 + p_{-i} - p_i)/2, & \text{if} \, |p_i - p_{-i}| \le d, \\ p_i, & \text{if } p_i < p_{-i} - d, \\ 0, & \text{if } p_i > p_{-i} + d, \end{cases}$$

Assume $\bar{p}_2$ is fixed

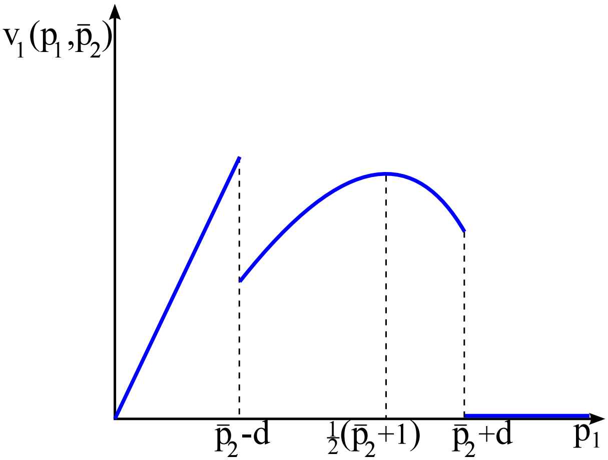

Fig.2

NE and ESS in the Hotelling game

Theorem (NE, Hotelling)

For $d \in [\frac{1}{2}, 1]$ *the unique NE is* $p_1^* = p_2^* = 1$. $\pi_1 = \pi_2 = 1/2$.

For $d = 0$ *the unique NE is* $p_1^* = p_2^* = 0$. $\pi_1 = \pi_2 = 0$.

For $d \in (0, \frac{1}{2})$ *NE does not exist.*

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Theorem (ESS, Hotelling)

For $d \in [\frac{1}{2}; 1]$ *the unique ESS is* $p_1^* = p_2^* = 1$. $\pi_1 = \pi_2 = 1/2$.

For $d \in [0; \frac{1}{2})$ *the unique ESS is* $p_1^* = p_2^* = 2d$. $\pi_1 = \pi_2 = d < 1/2$.

Simulation results, $d = 0.7$

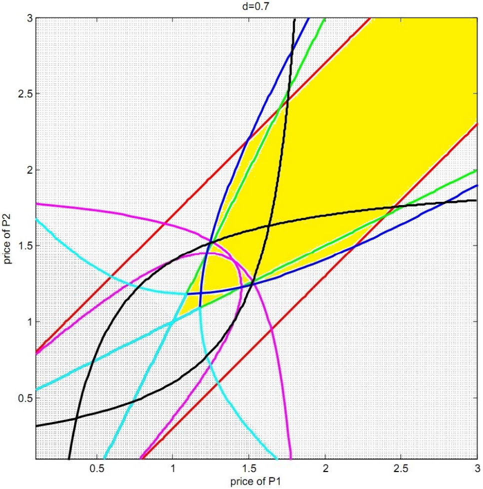

Fig.3a. (1, 1) is NE. Yellow area is NE-2.

Simulation results, $d = 0.5$

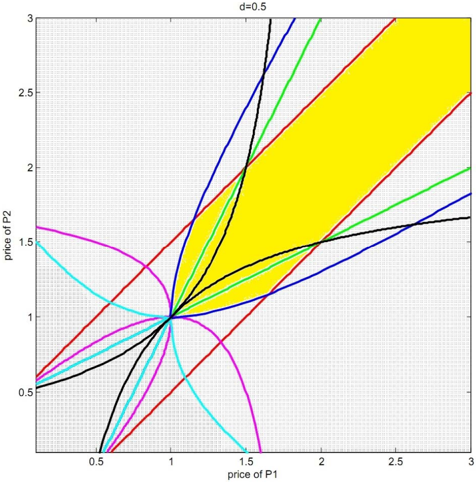

Fig.3b. (1, 1) is NE. Yellow area is NE-2.

Simulation results, $d = 0.35$

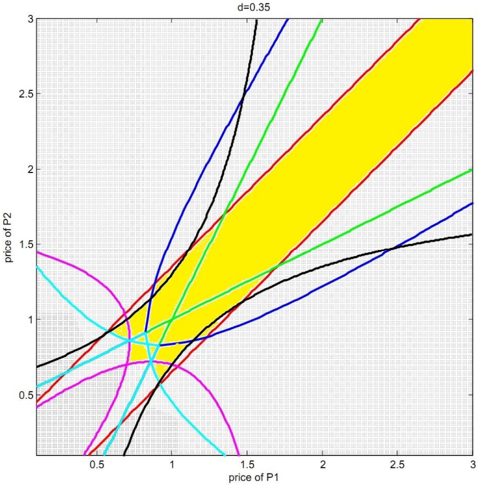

Fig.3c. (2d, 2d) is ESS. Yellow area is NE-2.

Simulation results, $d = 0.2$

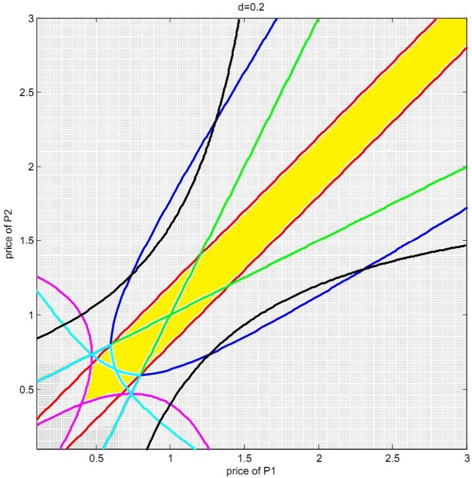

Fig.3d. (2d, 2d) is ESS. Yellow area is NE-2.

Boundary NE-2: a closed-form solution

Red: $|p_1 - p_2| = d$

Green: $p_1 = (p_2 + 1)/2$ and vice versa.

Pink: $2(p_1 - d) = p_2(1 + p_1 - p_2)$ and vice versa.

Dark blue: $p_1 = \frac{1+p_2}{2} + \sqrt{\left(\frac{1+p_2}{2}\right)^2 - 2d - p_2(1 - p_2)}$ and vice versa.

Light blue: $p_2 = \frac{1+p_1}{2} - \sqrt{\left(\frac{1+p_1}{2}\right)^2 - 2d - p_1(1 - p_1)}$ and vice versa.

Black: $p_2 = 2\left(1 - \frac{1-d}{p_1}\right)$ and vice versa.

Tullock contest

The model with two players

The contest success function translates the effort x of the players into the probabilities that each player will obtain the resource R.

$$p_i(x_i, x_{-i}) = \frac{x_i^\alpha}{x_i^\alpha + x_{-i}^\alpha}, \quad x \neq 0, i = 1, 2.$$

If $x = 0$ then $p_i = p_{-i} = 1/2$.

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The payoff function for each player

$$u_i(x_i, x_{-i}) = Rp_i(x_i, x_{-i}) - x_i.$$

Without loss of generality assume $R = 1$, $x_i \in [0, 1]$.

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When $\alpha > 2$ pure NE doesn't exist.

Simulation results, $\alpha = 0.7$

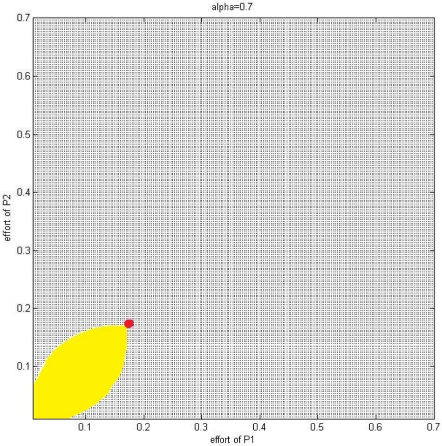

Fig.4a. Red point is NE, ESS, NE-2. Yellow area is NE-2.

Simulation results, $\alpha = 1.5$

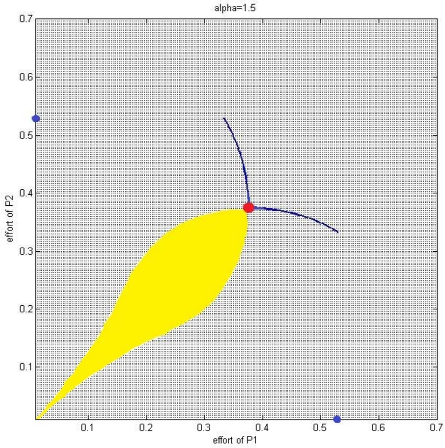

Fig.4b. Red point is NE, ESS, NE-2.
Blue curve and points are ESS, NE-2. Yellow area is NE-2.

Simulation results, $\alpha = 2.3$

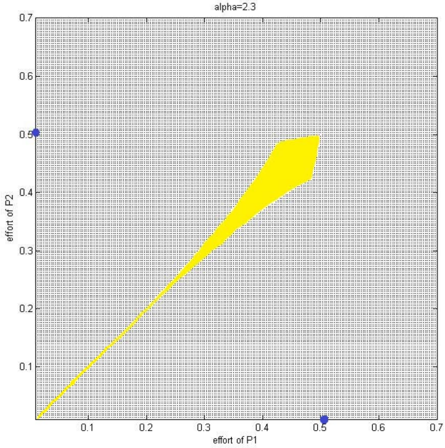

Fig.4c. Blue points are ESS, NE-2. Yellow area is NE-2.

Thank you for your attention!

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